NOISE INTENSITY IN THE FIELD OF A SUBSONIC TURBULENT JET

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Results are shown of digital-computer calculations pertaining to the acoustic properties of subsonic jets, by a method proposed earlier in [1, 2]. The gasdynamic and turbulence parameters of a jet, needed for calculating the acoustic radiation, have been determined beforehand.

A method has been proposed in [1, 2] for calculating the noise intensity in the acoustic field of a subsonic turbulent jet. An expression for the noise intensity at any point in the field of a subsonic isothermal jet of an ideal gas can, on the basis of [1, 2], be written as

$$I(\vec{r}_0) = \frac{\rho_0}{4\pi^2 a_0} \iiint\limits_W \left(\frac{\partial U}{\partial r}\right)^2 (\delta t)^2 \frac{\overline{v'^2}}{U_c^2} \frac{\omega_m^4}{\Phi^3} \left(\frac{1}{\Phi} - \frac{\omega_s}{\omega_m}\right)^2 \Omega \frac{dW}{|\vec{r}_0 - \vec{r}|^2}, \tag{1}$$

where, according to [2], the directivity factor Φ is given by the formula

$$\Phi = 1 - M_c \cos\alpha - \frac{\vec{n}}{a_0} \frac{\partial(\Delta\vec{\eta})}{\partial\tau}. \tag{2}$$

Here $\Delta\vec{\eta}$ is the radius vector between the point in a moving vortex region of a turbulent jet and the center of this region (the latter shifts during the motion of the vortex), and $\tau = t - |\vec{r}_0 - r|/a_0$.

According to (1), the noise intensity in the acoustic field of a jet is determined by the gasdynamic and turbulence parameters of that jet. Information about all these parameters can be found in [3-8] et al. The determination of some of them, however, requires a few additional stipulations.

Determination of Parameters δt and $\bar{\Phi}$. The intrinsic time lag should be defined as $\delta t = L/a$. Here $a = a_0/\bar{\Phi}$ denotes the velocity at which acoustic waves travel relative to the vortex region Ω, the latter moving at the convection velocity, and a is thus a function of the observation angle; $L = \sqrt{(\overline{L^2(\vec{r})})}$ denotes the mean-over-the-region distance from points in that region Ω to the sphere of radius $\vec{r}_0 - \vec{r}_c$. An estimate of this quantity based on simple geometrical considerations yields

$$\frac{L_r}{2\sqrt{2}} \leqslant \bar{L} \leqslant \frac{L_x}{2\sqrt{2}}.$$

Thus, the characteristic value of the intrinsic time lag can be expressed as

$$\delta t = \frac{k_1 L_x}{a_0 \bar{\Phi}}, \tag{3}$$

where

$$k_1 = \frac{\bar{L}}{L_x}, \quad \frac{1}{2\sqrt{2}} \frac{L_r}{L_x} \leqslant k_1 \leqslant \frac{1}{2\sqrt{2}}.$$

The following empirical relations for the integral scale factors of turbulence have been obtained in [3]:

$$L_x = 0.13x, \; L_r = 0.036x. \tag{4}$$

On the basis of the test curves in [4], it has been found that $L_x = 0.1x$ and $L_r = 0.045x$ is where the turbulent velocity fluctuations in a jet are maximum. With these relations, then, an approximate estimate for k_1 in

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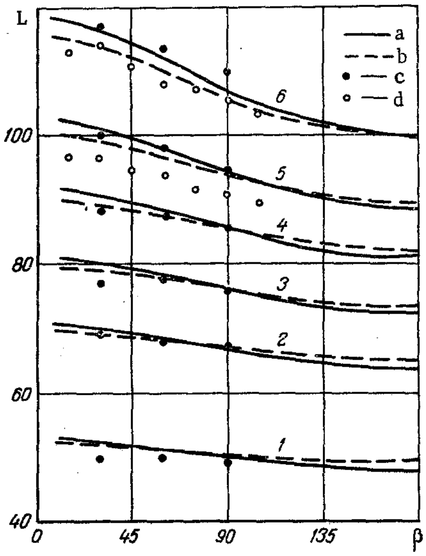

Fig. 1. Directivity characteristics of noise in jets with various values of the Mach number at $|\vec{r}_0| = 65r_a$: Ma = 0.12 (1), 0.223 (2), 0.312 (3), 0.435 (4), 0.590 (5), 0.900 (6): (a) calculated according to Eq. (12), (b) calculated according to the far-field formula, (c) test data according to [9], (d) results from [10] recalculated from $|\vec{r}_0| = 240r_a$ to $|\vec{r}_0| = 65r_a$. L (dB), β(°).

formula (3) would be $0.095 < k_1 < 0.353$.

The characteristic value of the directivity factor for a vortex region should, just as the characteristic value of δt, be defined in terms of the rms value of Φ over region Ω (inasmuch as the mean value of $\Delta\vec{\eta}$ in region Ω and the mean value of δt are both equal to zero). With (2) taken into account, we have

$$\bar{\Phi} = \left[(1 - M_c \cos\alpha)^2 + \left(\frac{\omega_m L_n}{a_0}\right)^2\right]^{\frac{1}{2}}. \tag{5}$$

This expression is the same as the one derived by J. E. F. Williams in [5].

Volume of Vortex Region and Time Scale Factors of Turbulence. The characteristic volume of a vortex region is defined in terms of integral turbulence scale factors. No test data are available on the scale factors of tangential turbulence, but, inasmuch as radial and tangential fluctuations of velocity are of the same order of magnitude [4], one may estimate $L_\varphi \sim L_r$. Then, obviously,

$$\Omega = k_2 L_x L_r^2. \tag{6}$$

On the assumption that the shape of a vortex region is nearly ellipsoidal, it follows that $k_2 \sim 4$.

The characteristic time scale of turbulence, in coordinates moving at the convection velocity of vortices, has been defined in [3] on the basis of space-time correlation curves of velocity fluctuations in a jet. The following empirical relation for the characteristic angular frequency has been proposed in [3]:

$$\omega_m = \frac{1}{3}\frac{\partial U}{\partial r}, \tag{7}$$

and was used in this analysis.

It is well known that the characteristic frequency of velocity fluctuations is much higher in stationary coordinates than in coordinates moving at the convection velocity. The frequency in stationary coordinates ω_s should be defined, in the same way as ω_m, on the basis of the autocorrelation curve (as a reciprocal of the time within which the correlation in the given coordinates diminishes by a factor e). On the basis of correlations shown in [6], the acoustic characteristics were calculated here with $\omega_s = 3\omega_m$.

Average-Velocity Gradient and Velocity of Vortex Convection. The average-velocity profile of the boundary layer in the initial zone and in the transition zone of a jet, derived in [7] on the basis of test data from various sources, was approximated here as follows:

$$\frac{U}{U_a} = \mathrm{erf}\,(1.5\xi^2 + 0.4\xi),$$

where

$$\xi = -3.7z + 0.534, \quad z = \sigma\frac{r - r_a}{x}.$$

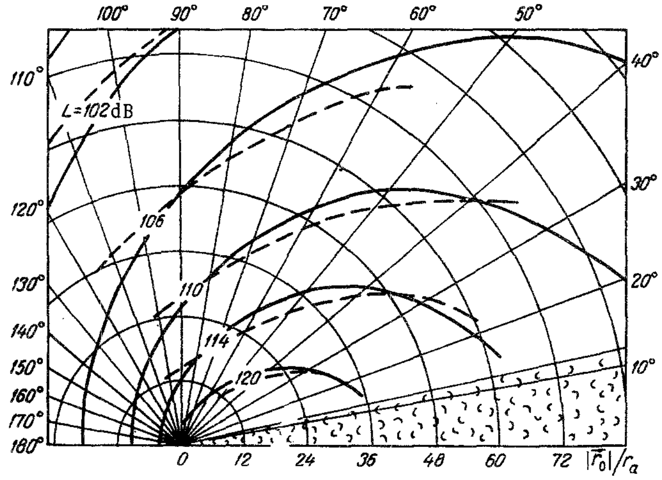

Fig. 2. Curves of equal noise levels in the acoustic field of a jet with Ma = 0.8: calculated according to formula (12) (solid lines), according to test data in [11] dashed lines).

From this follows an expression for the average-velocity gradient in the initial zone and in the transition zone:

$$\frac{\partial}{\partial r}\frac{U}{U_a} = -\frac{6.4\sigma}{\sqrt{\pi}x}(3\xi + 0.4)\exp[-(1.5\xi^2 + 0.4\xi)^2]. \tag{8}$$

Parameter σ accounts for the compressibility of a gas. According to the data in [8], it has been assumed here that

$$\sigma = 1 + 0.23M_a.$$

The test data in [7] pertaining to the average-velocity profile of the main zone, referred to velocity U_m at the jet axis, were approximated by the curve

$$\frac{U}{U_m} = \operatorname{erf}(\xi), \quad \xi = 37.1z^2 - 15.23z + 1.603, \quad z = \sigma\frac{r}{x}.$$

Considering that $U_m = U_a(x_t/x)$, we have for the average-velocity gradient in the main zone:

$$\frac{\partial}{\partial r}\frac{U}{U_a} = -\frac{20.7}{\sqrt{\pi}}\frac{\sigma x_t}{x^2}(7.18z - 1.473)\exp(-\xi^2). \tag{9}$$

In the main zone of a jet $U_c/U_m = 0.5$.

The velocity of vortex convection in the initial zone was measured in [3]. The following polynomial approximates the data:

$$\frac{U_c}{U_a} = 0.584 - 2.619z - 7.8z^2 + 52.7z^3.$$

Intensity of Turbulent Velocity Fluctuations. Information about the intensity of turbulent velocity fluctuations in a jet can be found in [3, 4, 7, 8] et al. The most complete data were given in [4], covering the results of multipoint measurements of fluctuating velocity components at 13 different jet sections for U_a = 100 m/sec. Representing the measured values in dimensionless coordinates, E. V. Vlasov [4] obtained universal curves for the fluctuating components in the initial zone as well as in the outer regions of both the transition and the main zone. The intensity of radial velocity fluctuations in these regions, where they can be described by universal curves, is defined as

$$\sqrt{\overline{v'^2}} = l_v\left|\frac{\partial U}{\partial r}\right|. \tag{11}$$

Expressions for the mixing length l_v are derived by comparing the results of calculations according to formulas (8), (9), and (11) with the results in [4]. For the inner region of the initial zone, where $-0.125 \le z \le 0$ and $0 \le x \le x_i$, it has been assumed that

$$l_v = x(2.70z^2 - 0.063z + 0.019).$$

For the outer regions of both the initial and the transition zone, where $0 \le z \le 0.144$ and $0 \le x \le x_t$,

$$l_v = x(-0.083z + 0.019).$$

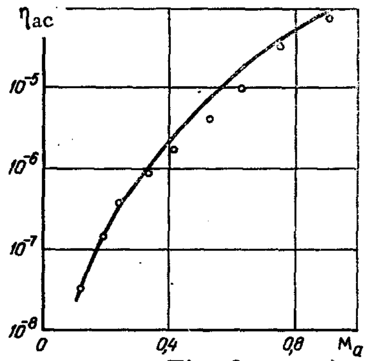

Fig. 3

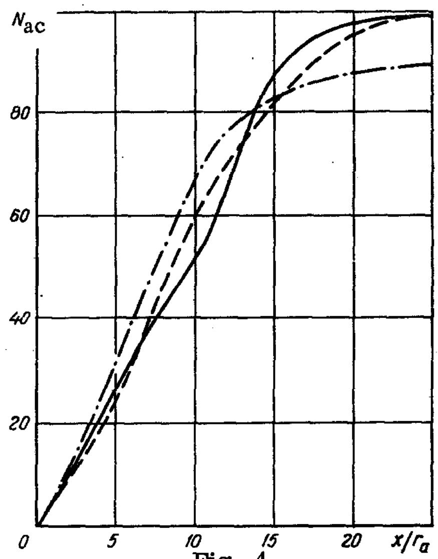

Fig. 4

Fig. 3. Acoustic-mechanical factor of a jet as a function of the Mach number: calculated according to formula (13) (curve), data from [9] (points).

Fig. 4. Relative acoustic power N_{ac} % of a jet segment of length x: calculated for Ma = 0.312 at $|\vec{r}_0| = 65r_a$ (solid line), according to data in [12] (dashed line), according to data in [13] (dashed-dotted line).

For the outer region of the main zone with the surface $z = 1/x_t$ regarded as the inner boundary, according to test data in [4],

$$l_v = x(-0{,}0665z + 0{,}0203);$$

and, moreover, $1/x_t \leq z \leq 0.22$, $x \geq x_t$. For the inner region of the main zone ($0 \leq z \leq 1/x_t$, $x \geq x_t$) the quantity $\sqrt{\overline{v'^2}}$ was considered constant over the section and equal to its value at $z = 1/x_t$. For the inner region of the transition zone, where $0 \leq r \leq r_a$ and $x_i \leq x \leq x_t$, the quantity $\sqrt{\overline{v'^2}}$ was approximated by the following relation linear with respect to x and r:

$$\sqrt{\overline{v'^2}} = [(0{,}1633 - 0{,}0132x)\,r + 0{,}0132x - 0{,}0553]\,U_a.$$

In our calculations we assumed $x_i = 8r_a$ and $x_t = 12r_a$.

Noise Levels in the Acoustic Field of a Jet. With the aid of relations (3)-(7), expression (1) for the noise intensity at any point in the acoustic field of a jet is transformed into

$$I(\vec{r}_0) = CK\rho_0 M_a^3 U_a^3 \iiint\limits_W \left(\frac{\partial}{\partial r}\frac{U}{U_a}\right)^6 \left(\frac{\sqrt{\overline{v'^2}}}{U_a}\right)^2 \left(\frac{1}{\Phi} - \frac{\omega_s}{\omega_m}\right)^2 \frac{x^5 r\,dr\,d\varphi\,dx}{\Phi^5\,|\vec{r}_0 - \vec{r}|^2}. \tag{12}$$

The value of the numerical coefficient C is determined from the factor in front of integral (1) and the experimental relations for L_r, L_x:

$$C = \frac{1}{324\pi^2}\,\frac{L_r^2 L_x^3}{x^5}.$$

The value of constant $K = k_1 k_2^2$ should be within the limits defined by the values of k_1 and k_2:

$$0{,}036 < K < 0{,}50.$$

The noise intensity in the acoustic field of a jet was calculated according to formula (12), with the aid of relations (8)-(11). The value of K was found by a comparison between calculated noise levels and test data in [9] for a jet with the Mach number Ma = 0.312 at $|\vec{r}_0| = 65r_a$ and $\beta = 90°$. This value K = 0.076 (for C = 0.63 $\cdot 10^{-9}$) concurs with the initial estimate for K. The calculated noise levels agree closely with the data in [9] over a wide range of Ma values. This is shown in Fig. 1, where noise levels at a circle of radius $|\vec{r}_0| = 65r_a$ have been calculated according to Eq. (12) with various values of the Mach number and also for the far field according to a formula derived from Eq. (12) by changing $|\vec{r}_0 - \vec{r}|$ to $|\vec{r}_0|$ and $\cos\alpha = (x_0 - x)/|\vec{r}_0 - \vec{r}|$ to $\cos\beta = x_0/|\vec{r}_0|$. It can be seen that both Eq. (12) and the far-field formula yield not very different directivity patterns of acoustic radiation, which agree with the test data in [9, 10], as the distance $|\vec{r}_0|$ decreases, the effect of the jet length on the directivity pattern becomes more pronounced.

The noise levels calculated at various distances in the acoustic field of a jet with Ma = 0.8 are shown in Fig. 2 and compared there with test values obtained in [11]. Since not the absolute rms values of pressure fluctuations were given in [11] but voltages recorded in millivolts, hence the test data from [11] had been plotted here in the form of equilevel curves. These curves were then superposed on the calculated curves at one field points with the coordinates $|\vec{r}_0| = 60r_a$ and $\beta = 90°$. The calculated noise levels were found to come close to the measured noise levels over the entire range of coordinates $|\vec{r}_0|$ and β covered in [11].

Acoustic Power in Various Zones of a Jet. The acoustic—mechanical factor of a jet is calculated according to the formula

$$\eta_{ac} = \frac{4|\vec{r}_0|^2 \int_0^{\pi} I(\beta)\sin\beta d\beta}{r_a^2 \rho_a U_a^3}. \tag{13}$$

This factor has been plotted in Fig. 3 as a function of the Mach number, and the acoustic component of radiation in the various jet zones can then be determined from Fig. 4. According to such calculations, 90% of the acoustic energy is radiated within a jet zone 12 diameters long beginning from the nozzle throat. Moreover, the initial zone and the transition zone, comprising the first jet segment 6 diameters long, radiate 60% of the total energy. The acoustic power per unit length is almost constant in these two zones, but decreases as $(x_t/x)^6$ in the main zone — according to formula (8) and along with $(\partial/\partial r)(U/U_a)^6$ under the integral in (12). Numerical computations have shown, at the same time, that acoustically most active is the jet region at the beginning of the main zone between sections $x = 12r_a$ and $x = 14r_a$, where 20% of the total energy is radiated. The high acoustic activity of this segment results from the intensity of turbulent mixing, which is maximum here, and from $(x_t/x)^6$ remaining close to unity throughout this segment.

The results of computations shown here refer to jets with the Mach number below unity. On the basis of this jet model, however, the range of the Mach number may be extended somewhat, if additional information about jet turbulence under such conditions is given. Formula (2) for calculating the noise intensity is not applicable to jets with the Mach number Ma 2 or higher, because it has been derived on the assumption of an ideal incompressible gas.

The method of calculation proposed in [1, 2] and here yields the noise intensity at any point in the acoustic field of a jet, except in the near field with predominating hydrodynamic fluctuations. A comparison between calculated and measured values indicates a satisfactory agreement at distances beyond 2-4 diameters away from the nominal jet boundary.

NOTATION

$\vec{r}$	is the radius vector;
x, r	are the cylindrical coordinates coupled to a jet with axial symmetry;
β	is the angle between vector $\bar{r}_0$ and the jet axis;
U	is the mean longitudinal velocity;
U_c	is the velocity of vortex convection;
v'	is the radial fluctuation component of velocity;
ρ	is the density of gas;
a	is the velocity of sound;
Ma	is the Mach number;
W	is the volume of boundary layer of a jet;
x_i	is the abscissa of the end of the initial zone;
x_t	is the abscissa of the end of the transition zone;
Ω	is the characteristic volume of the vortex region;
L_x, L_r, L	are the longitudinal, radial, and tangential integral scale factors of turbulence;
$L_n = \sqrt{1/2(L_x^2 + L_r^2)}$;	
ω_s, ω_m	are the characteristic frequencies in stationary and in moving coordinates respectively;
δt	is the characteristic intrinsic time lag;
$n = \vec{r}_0 - \vec{r}/|\vec{r}_0 - \vec{r}|$;	
$\cos\alpha = x_0 - x/|\vec{r}_0 - \vec{r}|$;	

U_m is the velocity at the jet axis;
$M_c = U_c/a_0$;
I is the noise intensity;
L is the intensity level (dB);
η_{ac} is the acoustic–mechanical factor.

Subscripts

0 refers to an unperturbed medium;
a refers to nozzle throat section;
c refers to center of vortex region;
Dash above a symbol indicates average value.

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